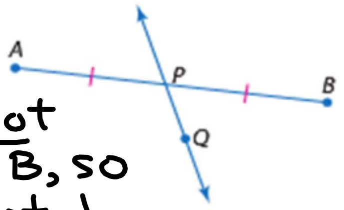
6-1

6-1 BISECTORS OF TRIANGLES

1 Perpendicular Bisectors

[redacted], you learned that a segment bisector is any segment, line, or plane that intersects a segment at its midpoint. If a bisector is also perpendicular to the segment, it is called a **perpendicular bisector**.

$\overline{PQ}$ is not $\perp$ to $\overline{AB}$, so $\overline{PQ}$ is not $\perp$ bisector.

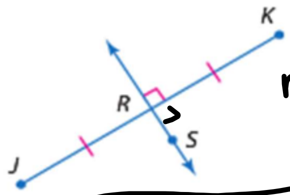


$\overleftrightarrow{PQ}$ is a bisector of $\overline{AB}$.

$\overline{AP} \cong \overline{PB}$

P is midpoint of $\overline{AB}$.

$m\angle SRK = 90$

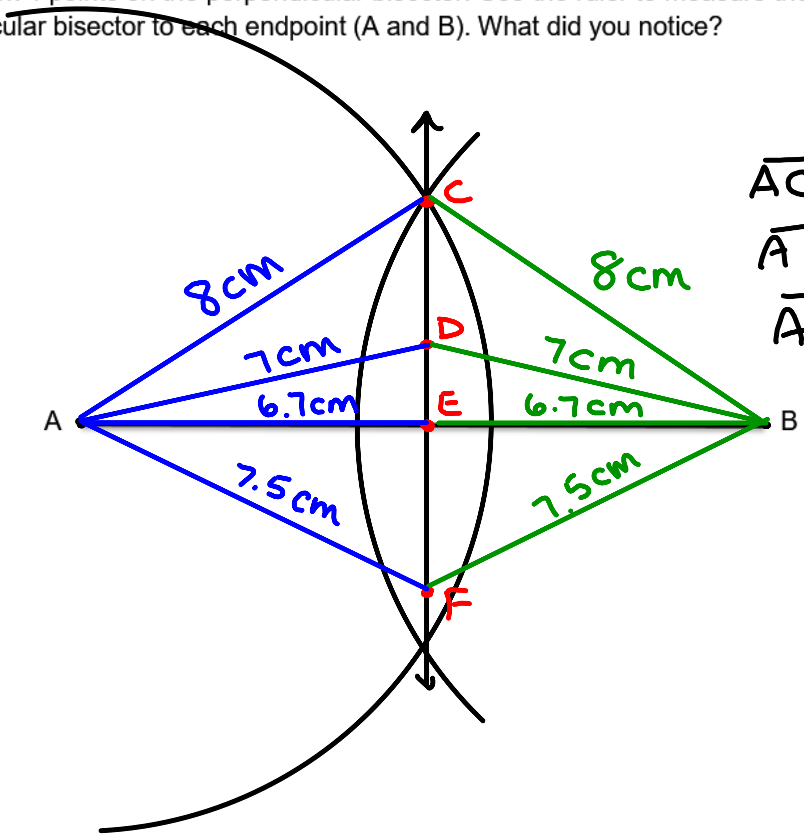


$\overleftrightarrow{RS}$ is a perpendicular bisector of $\overline{JK}$.

$\overline{JR} \cong \overline{RK}$

R is midpoint of $\overline{JK}$.

Take a compass and straightedge and construct the perpendicular bisector of segment AB below. Then, mark 4 points on the perpendicular bisector. Use the ruler to measure the distance from the perpendicular bisector to each endpoint (A and B). What did you notice?



$\overline{AC} \cong \overline{BC}$

$\overline{AD} \cong \overline{BD}$

$\overline{AE} \cong \overline{BE}$

$\overline{AF} \cong \overline{BF}$

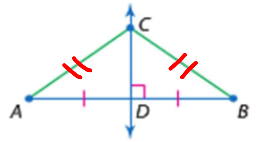
6-1

Theorems Perpendicular Bisectors

5.1 Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

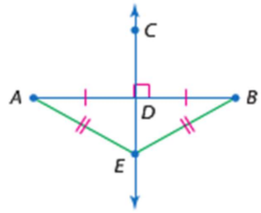
Example: If $\overline{CD}$ is a $\perp$ bisector of $\overline{AB}$, then $AC = BC$.



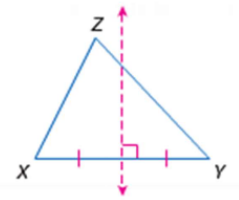
5.2 Converse of the Perpendicular Bisector Theorem

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

Example: If $AE = BE$, then E lies on $\overline{CD}$, the $\perp$ bisector of $\overline{AB}$.



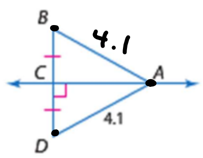
You should note that the perpendicular bisector of a side of a triangle does not necessarily pass through a vertex of the triangle. For example, in $\triangle XYZ$ below, the perpendicular bisector of $\overline{XY}$ does not pass through point Z.



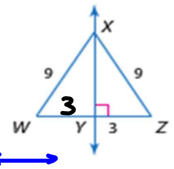
EXAMPLE 1: Find each measure.

HW

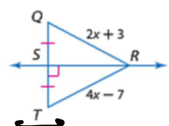
a. AB b. WY $\boxed{\text{c. RT}}$



$\overleftrightarrow{AC}$ is the $\perp$ bisector of $\overline{BD}$; C is the midpoint of $\overline{BD}$; $\overline{BC} \cong \overline{CD}$; A is equidistant from B and D $\rightarrow$ $\overline{AB} \cong \overline{AD}$.



$\overleftrightarrow{XY}$ is the $\perp$ bisector $\overline{WZ}$; Y is the midpoint of $\overline{WZ}$ $\rightarrow$ $\overline{WY} \cong \overline{YZ}$.



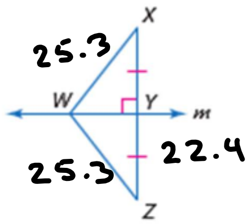
$\overleftrightarrow{SR}$ is the $\perp$ bisector of $\overline{QT}$; S is the midpoint of $\overline{QT}$; $\overline{QS} \cong \overline{QT}$. R is equidistant from Q and T $\rightarrow$ $\overline{QR} \cong \overline{RT}$.

$$2x+3 = 4x-7$$
$$-2x+7 \quad -2x+7$$
$$\frac{10}{2} = \frac{2x}{2}$$
$$5 = x$$

$$RT = 4x-7$$
$$4(5)-7$$
$$20-7$$
$$13$$

6-1

EXAMPLE 2: Use the picture to the right to find the following.



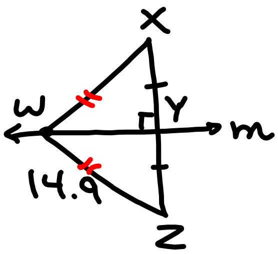
a. If WX = 25.3, YZ = 22.4, and WZ = 25.3, find XY.

$\overleftrightarrow{WY} = \perp$ bisector of $\overline{ZX}$.

Y = midpoint of $\overline{ZX}$.

$\overline{ZY} \cong \overline{YX}$

$\boxed{XY = 22.4}$

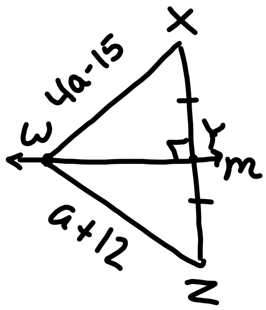


b. If m is the perpendicular bisector of XZ and WZ = 14.9, find WX.

line m

$\overline{WX} \cong \overline{WZ}$ by $\perp$ bisector thm.

$\boxed{WX = 14.9}$



c. If m is the perpendicular bisector of XZ, WX = 4a – 15, and WZ = a + 12, find WX.

$\overline{WX} \cong \overline{WZ}$ by $\perp$ bisector thm. →

$$4a - 15 = 1a + 12$$
$$-1a + 15 \quad -1a + 15$$
$$\frac{3a}{3} = \frac{27}{3}$$
$$a = 9$$

$WX = 4(9) - 15$

$36 - 15$

21